

Resource Allocation for D2D Communication in Cellular Networks Based on Stochastic Geometry and Graph-coloring Theory

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Abstract

In a device-to-device (D2D) underlaid cellular network, there exist two types of co-channel interference. One type is inter-layer interference caused by spectrum reuse between D2D transmitters and cellular users (CUEs). Another type is intra-layer interference caused by spectrum sharing among D2D pairs. To mitigate the inter-layer interference, we first derive the interference limited area (ILA) to protect the coverage probability of cellular users by modeling D2D users' location as a Poisson point process, where a D2D transmitter is allowed to reuse the spectrum of the CUE only if the D2D transmitter is outside the ILA of the CUE. To coordinate the intra-layer interference, the spectrum sharing criterion of D2D pairs is derived based on the (signal-to-interference ratio) SIR requirement of D2D communication. Based on this criterion, D2D pairs are allowed to share the spectrum when one D2D pair is far from another sufficiently. Furthermore, to maximize the energy efficiency of the system, a resource allocation scheme is proposed according to weighted graph coloring theory and the proposed ILA restriction. Simulation results show that our proposed scheme provides significant performance gains over the conventional scheme and the random allocation scheme.

Keywords: Device-to-Device (D2D), Spectrum Reuse, Interference, Stochastic Geometry, Weighted Graph-Coloring

1. Introduction

Growing demands for providing high data rate and low energy cost to support the rapidly increased mobile data traffic have been a great impetus to the development of novel technologies of improving system capacity [1-6]. Device-to-device (D2D) communication has drawn much attention due to its short mutual distance and high spectrum reuse gain by allowing direct communications between two or more user equipment in proximity [2]. As a result, D2D has been considered in the fifth generation (5G) as an underlay to cellular networks [1].

On the other hand, the energy efficiency (EE) of the system is becoming mainstream pursuit with the growth of energy demand and the rise of energy prices [4]. A D2D network underlying a cellular network can improve the spectrum efficiency (SE) and energy efficiency (EE) of the system [5-6], and reduce the traffic load of the macro BS [3]. However, resource sharing among cellular users and D2D users will cause serious co-channel interference.

To solve this problem, there has been extensive research on design and analysis of resource allocation schemes for D2D underlaid cellular networks [7-16]. Authors in [7] derived the average coverage probability of cellular users with corresponding density of potential D2D users based on stochastic geometry theory. In [8], a distance-based power control scheme was devised to enhance SE and coverage probability of users. The authors in [9] proposed a resource allocation and power control algorithm by exploiting the properties of fractional programming and penalty function to maximize the EE of DUEs, while the number of DUEs is less than that of CUEs. A two-loop iterative algorithm was designed in [10] to solve the energy efficiency fairness problem with the constraint of the maximum transmit power and the minimum data rate requirement. Yang et al. [11] investigated the energy-efficient power control problem for DUEs to maximize the total EE and the individual EE. The authors in [12] proposed a resource allocation algorithm which aimed to maximize the minimum weighted energy efficiency of DUEs as well as satisfy the minimum data rates of CUEs. In [13], a reverse iterative combinatorial auction (ICA) game was formulated to improve the system EE, while the QoS requirements of users were ignored. In [14], the authors formulated two resource allocation problems to optimize system EE and total individual EE considering the QoS of CUEs. An energy-efficient scheduling scheme among DUEs was proposed in [15], while only the QoS of DUEs was considered. A user grouping and power control scheme were proposed in [16] to maximize the system EE, while it only considered the impact of interference on DUEs when grouping, which may cause serious interference to CUEs.

Apart from these high complexity optimizing methods, the interference limited method was considered in [17-18] to reduce the computational complexity. A δ_D -interference limited area (ILA) scheme was designed to reduce the interference from CUEs to D2D users in [17]. The authors in [18] proposed a minimum distance restriction to alleviate the interference from cellular users to D2D links reusing the same resources. However, these are generally designed to improve SE. Different from the exiting work, we consider the EE programming problem based on the QoS constraints of both CUEs and DUEs based on stochastic geometry modeling and weighted graph coloring to mitigate co-channel interference in D2D underlaid cellular networks.

In this paper, a resource allocation scheme in D2D underlaid cellular networks was proposed, where D2D transmitters reuse the uplink spectrum of CUEs. Resource reusing can improve the energy efficiency of the system but will cause serious interference to CUEs. To guarantee the QoS of CUEs, an interference limited area (ILA), which ensure that the SINR of the CUE

be greater than a specified threshold with a large probability, is derived for CUEs based on the stochastic analysis. That is to say, the DUE who is out of ILA of the CUE is allowed to reuse the CUE's resource. Otherwise, resource sharing between the DUE and CUE is forbidden. Then, based on the above restrictions, we formulated a user grouping scheme that meet the DUE's distance restriction to avoid the DUEs suffer from serious interference. Finally, we propose an optimal resource selection scheme to maximize system EE based on the weighted graph-coloring theory. The main contributions in this paper are summarized as follows.

1) We propose a resource reuse scheme, which allows multiple D2D users reuse cellular user's spectrum resource, without increasing the overhead of conventional cellular system based on the stochastic geometry theory. And the energy efficiency of system can be greatly enhanced.

2) To depress the D2D-to-cellular interference, we propose an interference limited area for CUEs by modeling the D2D locations as a Poisson point process. Compare to the scheme in [16], the proposed scheme can efficiently protect the CUE's performance from serious interference.

3) In order to reduce the intra-layer interference among DUEs, we formulate a user grouping scheme based on the limitation of user distance. Based on the result of grouping scheme, we select an optimal DUE according to the interference graph to reuse the resource.

4) We demonstrate the effectiveness of our proposed resource reuse scheme by simulations.

The rest of the paper is organized as follows. In Section 2, we describe the system model and problem formulation to improve EE for D2D underlaid cellular networks. Section 3 describes the proposed resource allocation scheme based on ILA and the weighted graph coloring theory. The simulation results are given in Section 4. And we draw the conclusions in Section 5. Proof of the Theorem is given in the Appendix.

2. System Model and Problem Formulation

2.1 System Model

We consider an uplink scenario in a single-cell cellular system, in which there exist two kinds of communications, as illustrated in Fig. 1. One is called cellular communication. Another is D2D communication which is an underlay to the cellular communication. In the system, M cellular user equipment (CUE) and K D2D user equipment (DUE) ($M \leq K$) are randomly distributed in the coverage area of the base station (BS). DUEs are distributed according to the Poisson point process (PPP) Φ of intensity λ_d . Each D2D pair consists of a D2D transmitter (DT) and a D2D receiver (DR) with a fixed distance R_D . There are M orthogonal spectrum resources allocated to M CUEs respectively. Without loss of generality, we assume that the i -th resource block is allocated to the i -th CUE. DUEs share the same resource set with CUEs. Therefore, the co-channel interference consists of three parts: 1) interference received by BS from DT; 2) interference received by DR from CUE; 3) interference received by DR from other DT sharing the same spectrum resource with that D2D pair.

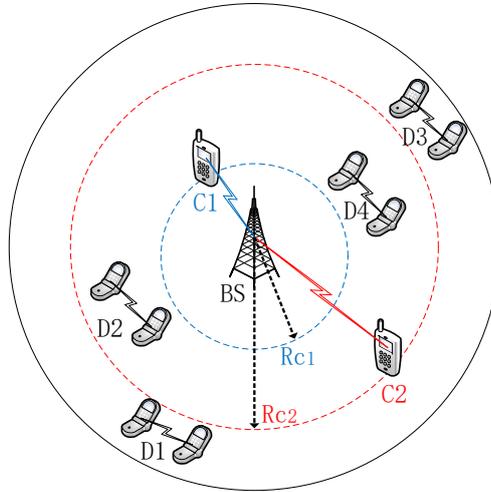


Fig. 1. System model and the illustration of resource allocation

In the following, we use $C_i(i=1, 2, \dots, M)$ and $D_j(j=1, 2, \dots, K)$ to denote the i -th CUE and j -th D2D pairs respectively; DT_j , and DR_j represent the transmitter of D_j and the receiver of D_j respectively. The channel gain can be expressed as

$$g = h \cdot \beta \tag{1}$$

where h represents the small scale fading, $h \sim \exp(\mu)$, and $\beta = d^{-\alpha}$ is the large scale fading, in which d is the distance between transmitter and receiver; α is the path loss exponent. The useful symbol information is summarized as **Table 1**.

Table 1. The Implication of Symbols

Symbols	Implication
g_i^C	channel gain between C_i and BS
g_j^D	channel gain between DT_j and BS
g_{ij}^{cd}	channel gain between C_i and DR_j
g_{kj}^{dd}	channel gain between DT_k and DR_j
$d_{C_i, B}$	distance from C_i to BS
$d_{D_j, B}$	distance from DT_j to BS
d_{C_i, D_j}	distance from C_i to DR_j
d_{D_k, D_j}	distance from DT_k to DR_j
$\beta_{C_i, B}$	large scale fading between C_i and BS
$\beta_{D_j, B}$	large scale fading between DT_j and BS
β_{C_i, D_j}	large scale fading between C_i and DR_j
β_{D_j, D_k}	large scale fading between DT_j and DR_k

The received Signal to Interference plus Noise Ratio (SINR) at the BS from C_i and the SINR of D_j using the i -th resource block can be represented as

$$SINR_i^C = \frac{P_c g_i^C}{\sum_{j=1}^K \pi_{ij} P_d g_j^D + N_0} \tag{2}$$

$$SINR_j^{D(i)} = \frac{P_d g_{jj}^{dd}}{P_c g_{ij}^{cd} + \sum_{k \neq j}^K \pi_{ik} P_d g_{kj}^{dd} + N_0} \quad (3)$$

where P_c , and P_d is the transmit power of CUE and DT respectively; N_0 is the power of the additive white Gaussian noise; π_{ij} is a resource reuse metrics. $\pi_{ij} = 1$ indicates that D_j shares the resources of C_i , while $\pi_{ij} = 0$ indicates that C_i and D_j are not allowed to share the same resources. According to SINR in (2) and (3), we can write the energy efficiency (EE) of C_i as

$$EE_i^C = \frac{\log_2(1+SINR_i^C)}{P_c + P_{cir}} \quad (4)$$

where P_{cir} represent the circuit power consumption of user. Similarly, the energy efficiency (EE) of D_j can be expressed as

$$EE_j^D = \frac{\log_2(1+SINR_j^D)}{P_d + 2P_{cir}} \quad (5)$$

2.2 Problem Formulation

The EE of the system can be expressed as

$$\begin{aligned} EE_{sum} &= \sum_{i=1}^M EE_i^C + \sum_{j=1}^K EE_j^D \\ &= \sum_{i=1}^M \frac{\log_2(1+SINR_i^C)}{P_c + P_{cir}} + \sum_{i=1}^M \sum_{j=1}^K \pi_{ij} \frac{\log_2(1+SINR_j^{D(i)})}{P_d + 2P_{cir}} \end{aligned} \quad (6)$$

We aim to maximize the system EE with the SINR constrain of CUEs and DUEs, i.e.,

$$\max. EE_{sum} \quad (7)$$

$$\text{s.t. } SINR_i \geq SINR_c \quad \forall C_i \in \mathcal{C} \quad (7a)$$

$$SINR_j^{D(i)} \geq SINR_d \quad \forall D_j \in \mathcal{D} \quad (7b)$$

$$\sum_{i=1}^M \pi_{ij} \leq 1 \quad \forall j \in \{1, \dots, K\} \quad (7c)$$

where constraints (7a) and (7b) guarantee the QoS of CUE and DUE respectively, and constraint (7c) indicates that D_j is only allowed to reuse at most one cellular user's resource at each time. However, in the optimization problem (7), the number of spectrum allocation schemes for K D2D pairs is as huge as M^K . Hence, we try to obtain a low complexity scheme based on the stochastic geometry and graph coloring.

3. The Proposed Resource Allocation Scheme

In this section, we proposed a two-step resource allocation scheme to mitigate the inter-layer interference between CUEs and DUEs and intra-layer interference among users. Firstly, we derive an ILA based on stochastic geometry theory, where only DUEs outside the ILA of the CUE are allowed to reuse the spectrum with the CUE. Then, based on the ILA, we propose a multi-user resource sharing solution according to the interference graph.

3.1 ILA of Cellular Users

Since the cellular users are the primary users of cellular systems, we formulate an ILA to guarantee the QoS of the CUEs where DUEs outside the ILA are allowed to reuse the spectrum of the CUE.

The definition of the ILA can be thought of equivalently as the coverage probability of the CUEs interfered by a transmitter outside A_0 should be large enough [19]. Therefore, the ILA of CUEs can be derived by the coverage probability defined as $P(\text{SINR} > T)$. The design of ILA must, therefore, ensure that the SINR of the CUE be greater than a specified threshold with a large probability β , such that

$$P(\text{SINR}_i^C > T) \geq \beta \quad (8)$$

Unlike the existing work where interference D2D pairs are placed deterministically on a regular grid, we model their location as a PPP process with intensity λ_d . Hence, the SINR in (2) can be rewritten as

$$\text{SINR}_i^C = \frac{P_c h_i d_{C_i,B}^{-\alpha}}{N_0 + P_d h_0 R_0^{-\theta} + \sum_{j \in \Phi \setminus A_0} P_d h_j d_{D_j,B}^{-\theta}} \quad (9)$$

where $P_c h_i d_{C_i,B}^{-\alpha}$ and $P_d h_0 R_0^{-\theta}$ are the received signal power and the interference power caused by the D2D users at a random distance R_0 with path loss exponent $\theta > 2$, respectively. $\sum_{j \in \Phi \setminus A_0} P_d h_j d_{D_j,B}^{-\theta}$ is the cumulative interference from D2D transmitters outside A_0 ; A_0 is an interference limited area with BS as the center and R_0 as its radius.

Lemma 1: ξ is a random variable with probability density function (p.d.f.) $f_\xi(x)$, $h \sim \exp(\mu)$, τ is a positive constant. Then, we have

$$P\left(\frac{h}{\xi} > \tau\right) = E_\xi e^{-\mu\tau\xi}$$

Proof:

$$\begin{aligned} P\left(\frac{h}{\xi} > \tau\right) &= E_\xi P(h > \tau\xi) \\ &= E_\xi \int_{\tau\xi}^{\infty} \mu e^{-\mu x} dx \\ &= E_\xi e^{-\mu\tau\xi} \end{aligned}$$

Theorem 1: The ILA of a tagged CUE is an inner region of a circle with the BS as the center and R_0 as the radius, where

$$R_0 \geq f^{-1}(\beta) \quad (10)$$

with

$$f(R_0) = \frac{\exp\left(-\frac{\mu T d_c^\alpha N_0}{P_c}\right)}{1 + \frac{T d_c^\alpha P_d R_0^{-\theta}}{P_c}} \exp\left(-\frac{\pi \lambda_d}{M} \left(\frac{T P_d d_c^\alpha}{P_c}\right)^{\frac{2}{\theta}} \int_{R_0^2 \left(\frac{P_c}{T P_d d_c^\alpha}\right)^{\frac{2}{\theta}}}^{R_c} \left(\frac{1}{1+u^2}\right)^{\frac{\theta}{2}} du\right) \quad (11)$$

and f^{-1} is the inverse function determined by f uniquely; R_c is the cell radius. Theorem 1 indicates that DUEs out of the ILA A_0 can share spectrum with the tagged CUE. The proof of

theorem 1 were given at the appendix.

3.2 Interference limited threshold of DUEs

After implementing the above step, we consider the intra-layer interference among users who share the same spectrum resource. Therefore, we introduce a mutual interference restrictions mechanism based on the distance.

Similar to [17], the Signal-to-interference ratio (SIR) restriction of D_j between other DUEs and CUEs can be expressed as follows

$$SIR_{D_j \leftarrow D_k} = \frac{R_D^{-\theta}}{d_{D_k, D_j}^{-\theta}} \geq \mu_d \quad (12)$$

$$SIR_{D_j \leftarrow C_i} = \frac{P_d R_D^{-\theta}}{P_c d_{C_i, D_j}^{-\theta}} \geq \mu_d \quad (13)$$

where μ_d is the minimum SIR restriction of DUEs; $SIR_{D_j \leftarrow D_k}$ represent the SIR from D_k to D_j , $SIR_{D_j \leftarrow C_i}$ represent the SIR from C_i to D_j . Thus the distance restriction of D_j between CUE and DUE can expressed as

$$d_{D_k, D_j} \geq R_D \mu_d^{1/\theta} \quad (14)$$

$$d_{C_i, D_j} \geq R_D \left(\frac{P_c}{P_d} \mu_d \right)^{1/\theta} \quad (15)$$

3.3 Optimal Resource Reuse Scheme Based on Graph-Coloring

After the above two steps, we can obtain the set of DUEs who can reuse the resources of C_i , and it can be expressed as $\Phi_i = \{D_j, \dots, (j \in 1, 2, \dots, K)\}$; and the set of CUEs that can share the resources with D_j can be expressed as $\Psi_j = \{C_i, \dots, (i \in 1, 2, \dots, M)\}$.

As depicted in Fig. 1, the protection area of cellular user C_1 and C_2 can be expressed as the circular area with the base station as their center, with R_{C_1} using blue line and R_{C_2} using red line as their radius, respectively. Considering the case that one DUE has more than one CUEs that can be chosen to share the spectrum. For example, D_1 is out of the ILAs of C_1 and C_2 . To maximize EE of the system, we formulate a metric to choose the best resource based on weighted graph-coloring [20]. The EE of C_i and D_j can be expressed as

$$EE = \frac{\log_2 \left(1 + \frac{\beta_{C_i, B}}{\beta_{D_j, B}} \right)}{P_c + P_{cir}} + \frac{\log_2 \left(1 + \frac{\beta_{D_j, D_j}}{\beta_{C_i, D_j}} \right)}{P_d + 2P_{cir}} = \log_2 \left(\left(1 + \frac{\beta_{C_i, B}}{\beta_{D_j, B}} \right)^{\frac{1}{P_d + 2P_{cir}}} \cdot \left(1 + \frac{\beta_{D_j, D_j}}{\beta_{C_i, D_j}} \right)^{\frac{1}{P_c + P_{cir}}} \right) \quad (16)$$

From (14), the total EE of C_i and D_j depends on

$$\eta_{C_i, D_j} = \left(1 + \frac{\beta_{C_i, B}}{\beta_{D_j, B}} \right)^{\frac{1}{P_d + 2P_{cir}}} \cdot \left(1 + \frac{\beta_{D_j, D_j}}{\beta_{C_i, D_j}} \right)^{\frac{1}{P_c + P_{cir}}} \quad (17)$$

If $P_C = P_d + P_{cir}$, the total EE of C_i and D_j is an increasing function of η_{C_i,D_j}

$$\eta_{C_i,D_j} = \frac{\beta_{C_i,B}}{\beta_{D_j,B}} + \frac{\beta_{D_j,D_j}}{\beta_{C_i,D_j}} \quad (18)$$

Hence, we choose the CUE who has the maximal η_{C_i,D_j} with D_j to share the spectrum of D_j .

Fig. 2 shows the interference between different type of users as follows:

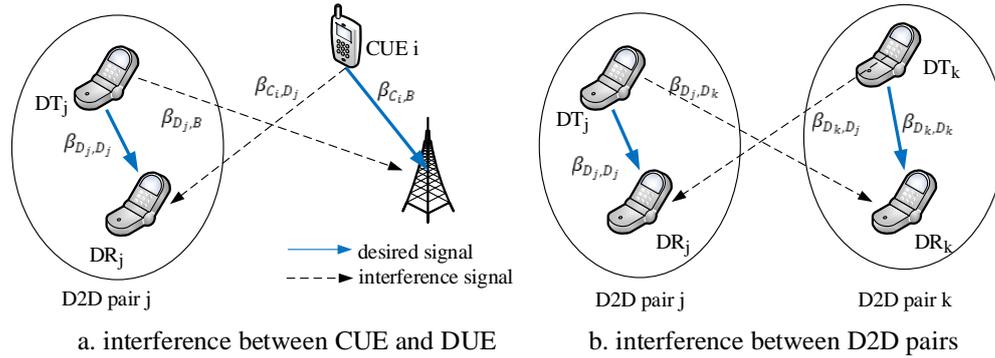


Fig. 2. Interference between different type of users

The proposed algorithm can be summarized as Algorithm 1.

Algorithm 1 Proposed resource allocation algorithm

- 1) Initialize the location of CUEs and DUEs according to Poisson point process;
- 2) BS get user's location information and calculate the channel gain from (1);
- 3) Initialize the reusable candidate DUEs for C_i : $\Phi_i = \{D_1, D_2, \dots, D_K\}$, candidate CUEs for D_j : $\Psi_j = \emptyset$, the optimal reusable DUEs for C_i : $\Phi_i^* = \emptyset$, the optimal reusable CUEs for D_j : $\Psi_j^* = \emptyset$;

Perform the resource allocation algorithm:

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for  $i \in C$ 
  for  $j \in D$ 
    if  $d_{D_j,B} > R_{C_i}$  and  $SIR_{D_j \leftarrow C_i} > \mu_d$ 
      for  $k \in \Phi_i \setminus j$ 
        if  $SIR_{D_j \leftarrow D_k} > \mu_d$ 
           $\Psi_j = \Psi_j \cup i$ 
        endif
      endfor
    else
       $\Phi_i = \Phi_i \setminus j$ 
    endif
  endfor
endfor
for  $j \in D$ 
   $i^* = \underset{i \in \Psi_j}{\operatorname{argmax}} \eta_{C_i,D_j}$ , select a CUE to reuse which has the maximal  $\eta$  with  $D_j$ 
   $\Psi_j^* = i, \Phi_i^* = \Phi_i^* \cup j$ , completed user grouping
endfor

```

4. Simulation Results and Analysis

To validate the proposed algorithm, we compare the algorithm with conventional algorithm in [16] and random algorithm by simulation. In the random algorithm, it performs the user grouping scheme the same as the proposed scheme while select reused resource randomly. The rest parameters of our simulation are summarized in Table 2.

Table 2. Simulation Parameters

Parameters	Values
cell radius $/R_C$	500m
power of CUEs/ P_C	25dBm
power of DUEs/ P_d	20dBm
circuit power/ P_{cir}	10mW
D2D link distance/ R_D	20m
noise power N_0	-114dBm
number of CUEs	10
density of DUEs/ km^2	20-100
path loss exponent α	4
path loss exponent θ	3
threshold γ_1, γ_2 [16]	10dB
SIR restriction of DUEs/ μ_d	10dB
simulation times	1000

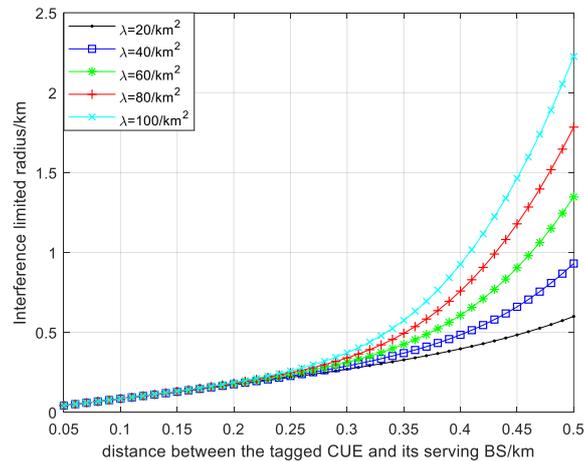


Fig. 3. Interference limited radius R_0 v.s. λ for different distance between the tagged CUE and its serving BS with $\beta=0.8$, and $T=-4$ dB.

Fig. 3 describes the interference limited radius for different distance between the tagged CUE and its serving BS. Simulation results in Fig. 3 show that the ILA of CUEs is influenced by the distance between CUE and BS and density of DUEs. The higher the distance is, the larger ILA is, and the size of ILA increases with the increasing of density of DUEs as well.

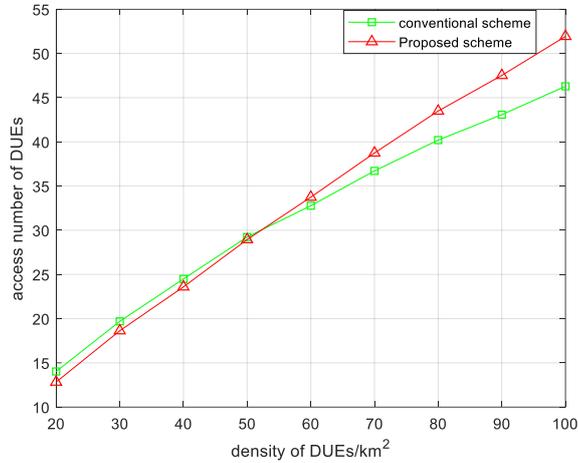


Fig. 4. Access number of DUEs for different density of DUEs

Fig. 4 gives the access number of DUEs for different density of DUEs. It shows that with the increasing of the total number of DUEs, more DUEs are allowed to access the system. As observed, the conventional scheme can access a little more users when the density of DUEs was low because it did not restrict for specific users. While when the density of users increases, the proposed scheme can access more DUEs to the system compared with conventional scheme because it does not allow users to access who may cause severe interference to others.

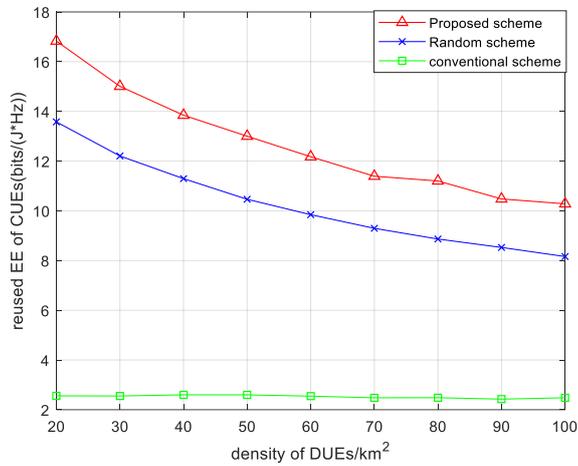


Fig. 5. Energy efficient of CUEs vs Different density of DUEs

As we can see in Fig. 5, compared with conventional scheme, the proposed scheme can avoid severe interference to the CUEs. The reason is that we formulate an ILA for CUE to prevent it suffering from serious interference. And the proposed scheme performs well compared to the random scheme because it carried out a graph coloring algorithm to coordination the interference between users.

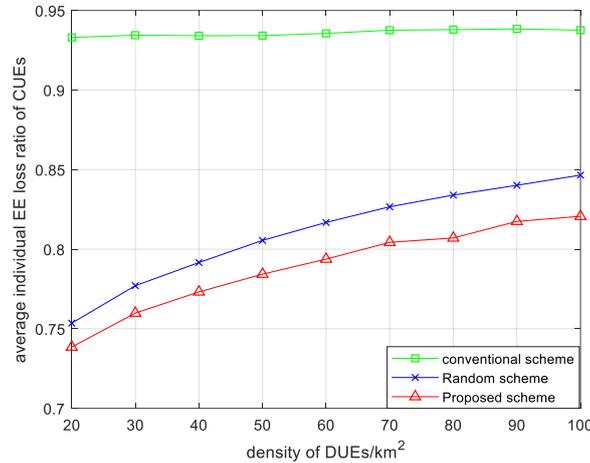


Fig. 6. Average EE loss ratio of CUEs vs Different density of DUEs

Fig. 6 displays the energy efficient loss ratio of CUEs with the different density of DUEs. It can be seen that the EE loss ratio of CUEs under the proposed scheme and random scheme is lower than the conventional scheme due to only CUEs with good channel conditions share spectrum to DUEs in our proposed scheme.

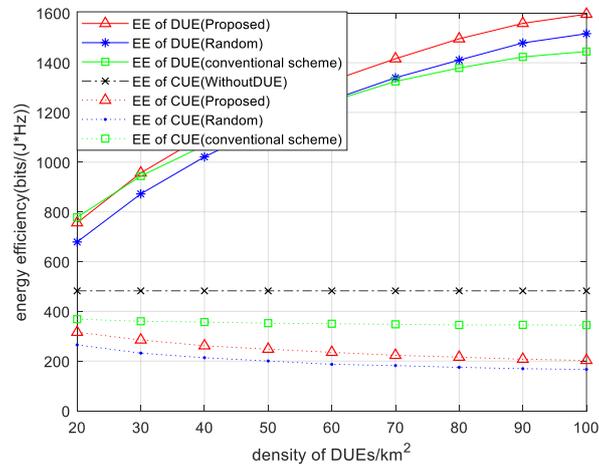


Fig. 7. Energy efficient of users vs Different density of DUEs

The energy efficiency of CUEs and DUEs are illustrated in Fig. 7. As observed, the EE of DUEs increases with the increasing of DUEs' density because the system can access more D2D pairs. And the EE of CUEs decreases with the increasing of DUEs' density. For the EE of DUEs, our proposed scheme performs well than the conventional scheme because it allows more DUEs to access, and it performs well than the random scheme due to it handles the interference problem using the theory of graph-coloring. For the EE of CUEs, contrast with the original system without DUEs, the proposed scheme decreases a bit more than the conventional scheme because the resource of the CUE who has better channel condition is

reused in the proposed scheme, and the proposed scheme avoids the resource of CUEs with worse channel condition from being reused.

5. Conclusion

In this paper, we consider an uplink D2D underlay scenario where the number of DUEs is larger than that of CUEs. In order to improve the access succeed rate of DUEs, we formulate a resource reuse scheme where the available spectrum resource of a CUE is allowed to be reused by multiple DUEs. To maximize the energy efficient under the restrictions, we drive the guard region for CUEs based on random point process to ensure its coverage probability. On the other hand, the performance of DUE can be improved by the ILA which avoids two DUE pairs with small distances sharing the same resource. Furthermore, we proposed an optimal reuse scheme according to the weighted graph coloring theory. Simulation results show that the proposed scheme can significantly improve the system energy efficient and avoid CUEs suffering from serious co-channel interference.

Appendix

In this Appendix, we will give the proof of Theorem 1.

Proof: The coverage probability can be described as

$$\begin{aligned}
& \mathbb{P}\left(\frac{P_C h_i d_C^{-\alpha}}{N_0 + P_C h_0 R_0^{-\theta} + \sum_{j \in \Phi \setminus A_0} P_d h_j d_j^{-\theta}} > T\right) \\
&= E\left(\exp\left(-\frac{\mu T d_C^\alpha}{P_C} (N_0 + P_d h_0 R_0^{-\theta} + \sum_{j \in \Phi \setminus A_0} P_d h_j d_j^{-\theta})\right)\right) \quad (a) \\
&= \frac{\exp\left(-\frac{\mu T d_C^\alpha N_0}{P_C}\right)}{1 + \frac{T d_C^\alpha P_d R_0^{-\theta}}{P_C}} E_\Phi\left(\prod_{j \in \Phi \setminus A_0} \frac{1}{1 + \frac{T d_C^\alpha P_d d_j^{-\theta}}{P_C}}\right) \quad (b) \\
&= \frac{\exp\left(-\frac{\mu T d_C^\alpha N_0}{P_C}\right)}{1 + \frac{T d_C^\alpha P_d R_0^{-\theta}}{P_C}} \exp\left(-2 \frac{\pi \lambda_d}{M} \int_{R_0}^{R_C} \left(1 - \frac{1}{1 + \frac{T d_C^\alpha P_d d_j^{-\theta}}{P_C}}\right) x dx\right) \quad (c) \\
&= \frac{\exp\left(-\frac{\mu T d_C^\alpha N_0}{P_C}\right)}{1 + \frac{T d_C^\alpha P_d R_0^{-\theta}}{P_C}} \exp\left(-2 \frac{\pi \lambda_d}{M} \int_{R_0}^{R_C} \left(\frac{1}{1 + \frac{P_C x^\theta}{T d_C^\alpha P_d}}\right) x dx\right) \\
&= \frac{\exp\left(-\frac{\mu T d_C^\alpha N_0}{P_C}\right)}{1 + \frac{T d_C^\alpha P_d R_0^{-\theta}}{P_C}} \exp\left(-\frac{\pi \lambda_d}{M} \left(\frac{T P_d d_C^\alpha}{P_C}\right)^{\frac{2}{\theta}} \int_{R_0^2 \left(\frac{P_C}{T P_d d_C^\alpha}\right)^{\frac{2}{\theta}}}^{R_C} \left(\frac{1}{1+u^2}\right) du\right) \quad (d) \\
&\triangleq f(R_0)
\end{aligned}$$

where (a) follows from Lemma 1. (b) follows from the independence of the small-scale fading powers. (c) follows from the probability generating functional [5] of the PPP. The integration limits are from R_0 to ∞ since the closest interferer is at least at a distance R_0 . (d) follows from a change of variables $v = (x(\frac{P_C}{T P_d d_C^\alpha})^{1/\theta})^2$. Notice that $f(R_0)$ is a monotonically increasing function. From (8) and (17), we obtain (10).

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